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Black hole emission rates and the AdS/CFT correspondence

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Abstract: We study the emission rates of scalar, spinor and vector particles from a 5 dimensional black hole for arbitrary partial waves. The solution is lifted to 6 dimensions, and the near horizon $BTZ \times S^3$ geometry of the black hole solution is probed to determine the greybody factors. We show that the exact decay rates can be reproduced from a $(1+1)$-dimensional conformal field theory which lies on the boundary of the near horizon geometry. The AdS/CFT correspondence is used to determine the dimension of the CFT operators corresponding to the bulk fields. These operators couple to plane waves incident on the CFT from infinity to produce emission in the bulk.

Keywords: D-branes, Black Holes in String Theory, Models of Quantum Gravity, Black Holes.

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Contents

1. Introduction

The study of the near horizon geometry of certain black holes has led to a better understanding of the origin of entropy and Hawking radiation from an underlying conformal field theory [1]–[5]. The near horizon geometry corresponds to anti-de Sitter space with or without certain identifications and is associated with a conformal field theory which lives on the boundary of the anti-de Sitter space. The CFT obtained thus carries non-trivial information about the black hole space-time. In this paper we concentrate on studying emission rates of particles from a five dimensional black hole and give a derivation of the rates using the conformal field theory which is associated with its near horizon geometry. The black hole solution is obtained by compactifying Type II B string theory on $T^4 \times S^1$. On retaining the S^1 as a compact direction with a large radius, it gives a black string solution wrapped around the S^1. The near horizon geometry of this configuration is $BTZ \times S^3$ where the BTZ black hole is 3-dimensional anti-de Sitter space with certain identifications [6]. We observe that the emission rates of neutral particles obtained in the black string background are the same as that from the 5-dimensional black hole [1, 7], and the near horizon BTZ geometry has a crucial role in determining the greybody factors [8, 9, 10]. We thus study the matter fields obtained as perturbations of the given 6-dimensional supergravity background and obtain the equations of motion of particles in the near horizon geometry, by considering a $AdS_3 \times S^3$ compactification of the six dimensional supergravity. Since BTZ space is locally AdS_3, to study the equation of motion of particles it suffices to study $AdS_3 \times S^3$ compactification. The particles in 6 dimension are expanded in terms of the harmonic functions on S^3, and higher partial wave objects appear as massive excitations on the BTZ spacetime. We separately study the behaviour of scalar, fermion and vector particles of the 6-dimensional $N = 8$ SUGRA spectrum. The fermions and vectors considered here are non-minimally coupled in six dimensions . We look at arbitrary partial waves for the particles and the greybody factors are calculated by studying the wave equations in the near horizon geometry and matching it suitably with the wavefunctions in asymptotically flat spacetime at a distance $r \sim l$ from the horizon, where l is the AdS_3 radius.

The equation of motion of a minimally coupled scalar near the black hole horizon reduces to the equation of motion of a massive scalar field in the BTZ background. By solving this equation of motion, as well as the equation of motion in six dimensions, far from the horizon, we match the wavefunctions at an intermediate region and determine the greybody factor. The latter agrees with the greybody factor obtained for arbitrary partial waves in [11]. The greybody factor calculation for non-minimally coupled fermions for arbitrary partial waves agrees with the result found previously in [12]. Here, the matching of the wavefunctions is non-trivial, as we have to solve the wavefunctions in three separate regions, near intermediate and far to obtain the greybody factor. The vector gauge fields have not been dealt with previously, and the emission rate calculation is thus a prediction for the five dimensional black hole. The near horizon $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ symmetry imposes some interesting restrictions on the one forms, which we exploit to obtain the solutions of the vector equations of motion.

Next we replace the entire near horizon geometry of black string solution by an effective 1+1-dimensional CFT which lies at a finite distance from the horizon, i.e. at $r \sim l \sim \sqrt{r_1 r_5}$. Here l is a measure of the size of the near horizon geometry, and r_1, r_5 are related to the charges of the black hole. A quantum mechanical calculation of the emission rate is done where a plane wave excites the operators of the CFT. The correlators of the CFT operators are determined by the AdS/CFT correspondence according to the prescription given in [13]–[20]. Unlike the other calculations of the emission rates, [10] where the AdS bulk solution couples to the operators, here it is the partial wave components of the *plane wave* which couple to the CFT operators and excite the CFT. The quantum mechanical calculation using the correlators with their proper normalisation constants reproduces the emission rates *exactly.*

In section 2, to fix our notation we review aspects of the six dimensional compactification to $BTZ \times S^3$. We determine the fermion and vector equations of motion on the three dimensional black hole background, and obtain the expression for the masses due to the orbital angular momentum of the particles. In section 3, the equations of motion for scalars, fermions and vectors are solved and the greybody factors are determined. In section 4 we determine the dimension of the operators which couple to the above particles and the corresponding correlators with the exact normalisation. The emission rates are then calculated by exciting the operators using plane waves which are incident on the CFT. Section 5 concludes with a discussion.

2. Five dimensional black holes and their near horizon geometry

The black hole solutions of string theory that we will consider arise from the low energy effective action of Type IIB string theory in 10-dimensions, by compactifying on $T^4 \times S^1$. The full 10-dimensional metric is given by [2, 21]

$$ds^2 = f_1^{-1/2} f_5^{-1/2} \left[-dt^2 + (dx^5)^2 + \frac{r_0^2}{r^2} \left(\cosh\sigma dt + \sinh\sigma dx^5 \right)^2 + f_1 \; dx_i dx^i \right] + \\ + f_1^{1/2} f_5^{1/2} \left[\left(1 - \frac{r_0^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega_3^2 \right], \tag{2.1}$$

where x^5 is along S^1 and x^i, $i = 6, \ldots, 9$ are the coordinates on the T^4. The functions f_1 and f_5 are given by:

$$f_1 = 1 + \frac{r_1^2}{r^2}, \qquad f_5 = 1 + \frac{r_5^2}{r^2}.$$

The resultant black hole metric in 5-dimensions after Kaluza-Klein reduction has six parameters, r_1, r_5, r_0, σ, V (volume of the T^4) and R (length of the S^1). In the case of the black hole obtained by wrapping Q_5 D-5 branes, Q_1 D-1 branes with momenta n along the 1-D brane the three charges of the black hole viz. Q_1, Q_5, n can be re-expressed as:

$$r_1^2 = \frac{gQ_1}{V}, \qquad r_5^2 = gQ_5, \qquad \frac{r_0^2 \sinh 2\sigma}{2} = \frac{g^2 n}{R^2 V}.$$

The black hole horizon is at r_0. The non-zero field strength in this background is given by:

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho} \frac{r r_1^2}{(r^2 + r_1^2)^2} (f_1 f_5)^{1/4}, \qquad H_{abc} = \epsilon_{abc} \frac{r_5^2}{r^3} (f_1 f_5)^{-3/4} \tag{2.2}$$

Where $\mu, \nu, \ldots$ run over t, x_5, r coordinates and a, b, c denote the angular directions. The metric (2.1) has the interesting property that in the near horizon limit $r \to r_0$ and in the so-called dilute gas approximation $r_1, r_5 \gg r_0, r_n$, it can be split up into three parts,

$$ds^2 = ds^2_{BTZ} + ds^2_{S^3} + ds^2_{T^4}, \tag{2.3}$$

where

$$ds^2_{BTZ} = -\frac{\Delta^2}{l^2\rho^2}dt^2 + \frac{l^2\rho^2}{\Delta^2}d\rho^2 + \rho^2\left(d\phi - \frac{\rho_+\rho_-}{l\rho^2}dt\right)^2$$
$$\Delta^2 = (\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2) \tag{2.4}$$

is the metric of a $(2+1)$-dimensional BTZ black hole, which is a solution of Einstein equation in 3-dimensions with a negative cosmological constant $\Lambda = -1/l^2$ [6]. We have made a coordinate change $r^2 = \rho^2 - \rho_-^2$ to get the above metric. The coordinate ϕ, the parameter l and the horizons of the BTZ black hole $\rho_\pm$ are related to the 5-dimensional black hole variables and parameters by the following relations:

$$\phi = \frac{x^5}{l}\,, \qquad \rho_+ = r_0\cosh\sigma\,, \qquad \rho_- = r_0\sinh\sigma\,, \qquad l^2 = r_1 r_5\,. \tag{2.5}$$

The part $ds^2_{T^4}$ is just the metric on the 4-torus and $ds^2_{S^3}$ is the metric on the three sphere with a constant radius l. The BTZ metric includes time, the periodic x^5 direction and the radial direction of the 5-dimensional black hole.

The above decomposition forms the basis of the approach we are considering, in which all thermodynamic properties of the black hole will be attributed to the 'non-trivial' BTZ part. Similar decompositions can be done in the case of black holes in other spacetime dimensions [22, 23]. The relevant near horizon part of the metric thus preserves $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ symmetry which is absent in the full five dimensional geometry. In fact as shown in earlier cases, the equation of motion of the particles in the five dimensional black hole background show this symmetry near the horizon. The inclusion of the extra direction x^5 does not affect this property, since the extra dimension is a Killing direction and does not change the symmetries of the equations of motion.

By compactifying the 10 D metric on T^4, the black string solution in 6D is probed. This is a solution of $N = 8$ supergravity in 6-dimensions. In $D = 6$, $N = 8$ supergravity theory, the spectrum consists of 40 fermions apart from 5 anti-self dual, anti-symmetric gauge fields, 16 vector fields, 25 scalar fields and 8 Rarita-Schwinger fields. Out of these, as seen in (2.2), only one of the anti-symmetric gauge field strength is non-vanishing apart from the metric background. There is a $\mathrm{SO}(5,5)$ global symmtery which gets broken due to the black hole background to $\mathrm{SO}(4) \times \mathrm{SO}(5)$. We look at certain particles in the spectrum, namely the minimally coupled scalars and non-minimally coupled fermions and gauge fields in 6D. The scalars correspond to gravitons along the T^4 direction. The scalars satisfy ordinary Klein Gordon equation in 6D, and on compactification on $AdS_3 \times S^3$, are expanded as $\phi = \sum \Phi(t,r,\phi)Y^{(L0)}$, where $Y^{(L0)}$ are spherical harmonics on S^3. The equation of motion for scalar fields for the partial wave L on S^3 satisfies the massive Klein-Gordon equation

$$[\Box - M^2]\Phi = 0 \tag{2.6}$$

in the AdS_3 spacetime with the mass μ given in terms of L as [24, 25]

$$M^2 = \frac{1}{l^2} L(L+2)$$

In the notation of [26] the fermion equation of motion is

$$\frac{i}{2}\Gamma^M D_M \chi^{a\alpha} - \frac{1}{16} P^a_{\dot{a}\ M} (\gamma^{\dot{a}})^\alpha_\beta \Gamma^M \Gamma^N \psi^\beta_{-N} - \frac{1}{24} F^{\dot{a}}_{MNP} \Gamma^{MNP} \gamma^{\beta\alpha}_{\dot{a}} \chi^a_\beta +$$
$$+ F^{\alpha\dot{\alpha}}_{MN} \left[(\gamma^a)_{\dot{\alpha}\dot{\beta}} \Gamma^{MN} \Gamma^P \psi^{\dot{\beta}}_{+P} - \frac{1}{4} \Gamma^{MN} \chi^{\dot{a}\beta}_{-} (\gamma_a)^\alpha_\beta (\gamma_{\dot{a}})^{\dot{\alpha}}_{\dot{\beta}} \right] = 0\,, \quad (2.7)$$

where $M, N, \ldots$ represent 6 dimensional world index, a and $\dot{a}$, SO(5) $\times$ SO(5) vector index, α, β SO(5) $\times$ SO(5) spinor index. The $+$ and $-$ sign denote the chirality of the fermions. In the above P are related to the kinetic term of the scalars, F^a_{MNP} is related to the three form field strength, $F^{\alpha\dot{\alpha}}_{MN}$ are related to the field strengths of the one form gauge fields. Now we study the compactification of this theory to $AdS_3 \times S^3$. From (2.2) it can be seen that that the non-zero background fields, the three form field strength is given in the near horizon limit by; $H^a_{\mu\nu\rho} = (1/l)\epsilon_{\mu\nu\rho}\delta^{a5}$ and $H^a_{bcd} = (1/l)\epsilon_{bcd}\delta^{a5}$ where $\mu, \nu, \ldots$ etc indicate the three AdS directions and $b, c, d \ldots$ are the S^3 directions. This gives the required equation $R_{\mu\nu\rho\lambda} = -1/l^2(g_{\mu\rho}g_{\nu\lambda} - g_{\nu\rho}g_{\mu\lambda})$ for the AdS_3 directions and $R_{bcde} = 1/l^2(g_{bd}g_{ce} - g_{cd}g_{be})$ for the S^3. Next, we factorise the fermion field χ in terms of an undetermined function of the BTZ coordinates, times the harmonic functions on the three-sphere. We also work in the representation where $(\gamma^5)\chi = \chi$. In this linearised approximation, the resultant expression is:

$$\Gamma^M D_M \chi - \frac{1}{12} H_{MNP} \Gamma^{MNP} \chi = 0\,. \quad (2.8)$$

The expansion in harmonics of S^3 is of the form $\chi = \sum \chi^{(p,\pm 1/2)} Y^{(p,\pm 1/2)}$, and obey $\nabla\!\!\!/ Y^{(p,\pm 1/2)} = \pm\imath(p+1) Y^{(p,\pm 1/2)}$, where p is a half integer, labeling the spin representation. Plugging in this expansion in the equation of motion (2.7), and using the decomposition of 6-dimensional Γ^M matrices into 3-dimensional ones as given in [25], the two-component equation takes the form

$$\gamma^\mu D_\mu \chi' + \frac{1}{l} \left(\mp(p+1) - 1 \right) \chi' = 0\,, \quad (2.9)$$

where $\chi^{(p,\pm 1/2)} = \chi'$, which can be written as:

$$\gamma^\mu \left(\partial_\mu + \omega_\mu + \frac{1}{l} \left[L + \frac{1}{2} \right] \right) \chi' = 0\,. \quad (2.10)$$

where $p = L + 1/2$, and we have chosen one of the eigenvalues of the spherical harmonic (choosing the other sign gives $L + 1/2 + 2$ for the mass term). The spin connections correspond to BTZ spacetime. Note that, here L stands for the orbital

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angular momentum, and in [9], the calculations were done for $L = 0$. From the above it also follows that the lowest mass term in the BTZ space time is non zero and equals 1/2. This is our basic set of equations for the determination fermionic of the greybody factor. It is interesting that on plugging the three dimensional spin connections and using the relations (2.5), it can be shown that this equation is the same as that of the fermionic fluctuations in the background of the 5-dimensional black hole in the near horizon limit [12]. We confine ourselves to particles without any Kaluza-Klein momentum along the compact direction x^5. In other words, the particles belong to the s-wave sector with respect to the BTZ black hole. Inclusion of the azimuthal quantum number along x^5 will imply charged fermion emission in five dimensions.

Similar decomposition can be made for the vector equations of $D = 6, N = 8$ supergravity into $AdS_3 \times S^3$. The exercise has been done in [27]. Note that this vector couples to the threeform, and hence its linearised equation of motion reduces to:

$$\nabla^M F^a_{MN} - \frac{1}{6}\epsilon_N^{\ PQRST}(\gamma_a)^\alpha_\beta F^\beta_{PQ} H^a_{RST} = 0\,. \tag{2.11}$$

The gauge fields when expanded in the spherical harmonics $A_\mu = \sum A_\mu^{(L,\pm1)} Y^{(L,\pm1)}$ reduces to:

$$\nabla^\nu \partial_{[\nu} A_{\lambda]} - \frac{1}{l}\epsilon_\lambda^{\ \nu\rho}\partial_{[\nu}A_{\rho]} = \frac{1}{l^2}L(L+2)A_\lambda\,, \tag{2.12}$$

where we have dropped the indices $(L, \pm 1)$. These set of equations correspond to a massive gauge field in the BTZ background, and we solve for this to get the required greybody factor.

3. Greybody factors

In this section, we solve the scalar, fermion and vector equations of motion of the previous sections to find the absorption cross-sections of the black hole for these particles. Since we study particles of various spins, a Newman-Penrose formalism would have been ideal for the study of particle propagation on the BTZ background. However, this has not been developed in three dimensions, and we separately consider the various equations of motion and find the solutions in the near horizon and in the asymptotic regions.

3.1 Scalar greybody factor

The scalar greybody factor for arbitrary partial waves for the five dimensional black hole was found in in [11]. Here, we exploit the near-horizon (BTZ) geometry of the black holes to solve the scalar wave equations. As stated before, the massless scalar wave equation for an arbitrary partial wave L in the $5D$ background can be reduced to the massive Klein-Gordon equation in the BTZ background. This equation was solved for the massless case in [8].

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From (2.6) and (2.4), we we get the massive s-wave scalar equation in BTZ background:

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\Delta^2}{l^2\rho}\frac{d\Phi}{d\rho}\right)+\frac{\omega^2 l^2\rho^2}{\Delta^2}\Phi - M^2\Phi = 0\,. \tag{3.1}$$

Defining

$$z=\frac{\rho^2-\rho_+^2}{\rho^2-\rho_-^2}$$

and assuming $\Phi(x^\mu)\sim e^{i\omega t}\Phi(\rho)$, the equation takes the form

$$z(1-z)\frac{d^2\Phi}{dz^2}+(1-z)\frac{d\Phi}{dz}+\left[\frac{A}{z}-B-\frac{M^2}{4(1-z)}\right]\Phi=0\,, \tag{3.2}$$

where

$$A=\left(\frac{\omega}{4\pi T_H}\right)^2, \qquad B=\left(\frac{\rho_-^2}{\rho_+^2}\right)A$$

and

$$T_H=\frac{\rho_+^2-\rho_-^2}{2\pi l^2\rho_+}$$

is the Hawking temperature of the BTZ black hole. Plugging in the ansatz

$$\Phi(z)=z^m(1-z)^n F[z] \tag{3.3}$$

we get

$$\begin{aligned} z(1-z)\frac{d^2F}{dz^2}+[(2m+1)-(2m+2n+1)z]\,\frac{dF}{dz}+ \\ +\left[\frac{m^2+A}{z}+\frac{n(n-1)-M^2/4}{1-z}-(m+n)^2-B\right]F=0 \end{aligned} \tag{3.4}$$

Setting the coefficients of the $1/z$ and the $1/(1-z)$ terms to zero, as required by the continuity with the solution very close to the horizon [28], the above equation reduced to the familiar hypergeometric equation

$$z(1-z)\frac{d^2F}{dz^2}+[(2m+1)-(2m+2n+1)z]\frac{dF}{dz}-[(m+n)^2+B]F=0\,. \tag{3.5}$$

Thus, the final solution is:

$$\Phi(z)=z^m(1-z)^n F[\alpha,\beta;\gamma;z] \tag{3.6}$$

where

$$\begin{aligned} &m=-i\sqrt{A}\,, && n=-\frac{L}{2} \\ &\alpha=-i\left(\sqrt{A}-\sqrt{B}\right)+n\,, && \beta=-i\left(\sqrt{A}+\sqrt{B}\right)+n \\ &\gamma=1-2i\sqrt{A} \end{aligned} \tag{3.7}$$

and we have substituted $M^2 = L(L+2)/l^2$. The flux of particles into the black hole can be calculated from the formula

$$\mathcal{F}_0 = \frac{2\pi}{i}\left[\frac{\Delta^2}{\rho}\Phi^*\frac{d\Phi}{d\rho} - \text{c.c.}\right] \tag{3.8}$$

which yields

$$\mathcal{F}_0 = 4\pi\omega l^2 \rho_+ \,. \tag{3.9}$$

Now to find the incoming flux at infinity, we have to solve the wave equation at very large distances from the black hole, where space time is almost flat. The corresponding wave equation is solved in the six dimensional black string background, with the metric given in eq. (2.1), with $r \to \infty$. The solution is expanded as $\sum \Phi(r) Y^{(L0)}$, where the $Y^{(L0)}$ are the spherical harmonics on S^3. Using $\nabla^2 Y^{(L0)} = -L(L+2)Y^{(L0)}$, where ∇^2 is the Laplacian on the S^3, the radial equation of motion follows:

$$\frac{1}{r^3}\frac{d}{dr}\left(r^3\frac{d\Phi}{dr}\right) + \left[\omega^2 - \frac{L(L+2)}{r^2}\right]\Phi = 0 \tag{3.10}$$

having the ingoing Bessel solution:

$$\Phi = \frac{1}{r}\left(A J_{L+1}(\omega r) + B N_{L+1}(\omega r)\right). \tag{3.11}$$

The asymptotic expansions of the Bessel functions yields the following flux at infinity:

$$\mathcal{F}_\infty = 2\left[|A|^2 + |B|^2 + i(A^*B - B^*A)\right]. \tag{3.12}$$

Since the far solution should smoothly go over to the near horizon (BTZ) solution, we investigate the nature of the solutions near the region $r \sim l$, till which region we assume that the AdS_3 geometry is a good approximation to the black hole spacetime. From eq. (2.5) and the dilute gas approximation, near $r \approx l \gg r_0 \sinh\sigma$, we get $\rho \sim r$ and hence the angular parts of the wavefunctions are the same. Thus, we simply compare the radial wavefunctions. The intermediate region is obtained by setting $z \to 1$ and $r\omega \ll 1$ in the hypergeometric and the Bessel solutions respectively to obtain the matching condition [29, 30]:

$$A = N^{-L/2}(L+1)(L!)^2\left(\frac{2}{\omega}\right)^{L+1}\frac{\Gamma(\gamma)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}\,. \tag{3.13}$$

where $N = \rho_+^2 - \rho_-^2 = r_0^2$. The other constant B is much smaller by factor of $(N\omega^2)^L$, and hence is neglected in the subsequent calculations.

A interesting point to note is that if we solve for the scalar wavefunction in the asymptotic AdS space, the solutions are obtained as $\Phi = J_{L+1}(\omega l^2/\rho) + N_{L+1}(\omega l^2/\rho)$, and thus at $\rho = r = l$, this has the exact polynomial behaviour as the flat space wave functions as the arguments of the Bessel functions both reduce to ωl. Although we

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do not use the scalar wave functions in asymptotically AdS_3 space to determine the greybody factor, it would be interesting to check whether the above observation has a deeper significance, since the location $r = l$ has no apparent physical significance.

The greybody factor is then evaluated using standard methods of calculation of absorption crossections by taking the ratio of the fluxes. Thus from (3.9), (3.12) and (3.13), the greybody factor is:

$$\begin{aligned}\sigma_{\text{abs}} &= \frac{4\pi}{\omega^3}(L+1)^2\frac{\mathcal{F}_0}{\mathcal{F}_\infty} \\ &= \frac{2\pi}{(L!)^4}\left(\frac{\omega}{2}\right)^{2L}\frac{N^{L+1}}{\omega}\sinh\frac{\omega}{2T_H}\left|\Gamma\left(1+\frac{L}{2}+\frac{i\omega}{4\pi T_-}\right)\Gamma\left(1+\frac{L}{2}+\frac{i\omega}{4\pi T_+}\right)\right|^2,\end{aligned} \tag{3.14}$$

where

$$\frac{1}{T_{-,+}} \equiv \frac{1}{T_H}\left(1-\frac{\rho_-}{\rho_+}\right)$$

and we have included the plane wave normalisation factor $\frac{4\pi}{\omega^3}(L+1)^2$. We have also used the identity $|\Gamma(1-ix)|^2 = \pi x/\sinh \pi x$. The above expression for the greybody factor reduces to the area of the black hole for $L = 0$, $T_- \gg T_+$, and $\omega \to 0$ [31].

3.2 Fermion greybody factor

We shall solve the equation of motion of the fermions equation (2.10) on the BTZ background, in a suitable set of coordinates. We define $\rho^2 = \rho_+^2\cosh^2\mu - \rho_-^2\sinh^2\mu$ and $x^\pm = \pm\rho_\pm t/l \mp \rho_\mp\phi$ and assume the following form of the wavefunctions:

$$\chi'_{1,2} = \frac{e^{-i(k^+x^+ + k^-x^-)}}{\sqrt{\cosh\mu\sinh\mu}}\psi_{1,2}\,,$$

where $(1,2)$ refer to the two components of the spinor. The spin connections for the BTZ-metric are:

$$\omega_{x^+} = -\frac{1}{2l}\cosh\mu\sigma^{01}\,, \qquad \omega_{x^-} = \frac{1}{2l}\sinh\mu\sigma^{21}$$

The equation of motion for ψ takes the following form:

$$\gamma^1\partial_\mu\psi + \gamma^0\frac{ilk^+}{\sinh\mu}\psi + \gamma^2\frac{ilk^-}{\cosh\mu}\psi + \left(L+\frac{1}{2}\right)\psi = 0\,. \tag{3.15}$$

Here we work in the representation, $\gamma^1 = \sigma^1, \gamma^0 = \imath\sigma^2, \gamma^2 = \sigma^3$. Then we define a new set of wavefunctions $\psi'_{1,2}$ as

$$\psi_1 + \psi_2 = \left(1-\tanh^2\mu\right)^{-1/4}\sqrt{1+\tanh\mu}\,(\psi'_1 + \psi'_2) \tag{3.16}$$

$$\psi_1 - \psi_2 = \left(1-\tanh^2\mu\right)^{-1/4}\sqrt{1-\tanh\mu}\,(\psi'_1 - \psi'_2)\,, \tag{3.17}$$

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whence the Dirac equation assumes the form:

$$(1-y^2)d_y\psi_2' - i\left(\frac{k^+}{y}+k^-y\right)\ \psi_2' = -[L+1-i(k^+ + k^-)]\psi_1' \tag{3.18}$$

$$(1-y^2)d_y\psi_1' + i\left(\frac{k^+}{y}+k^-y\right)\ \psi_1' = -[L+1+i(k^+ + k^-)]\psi_2' \,, \tag{3.19}$$

where we have defined $y = \tanh\mu$. Next, we choose the following ansatz

$$\psi_{1,2}' = B_{1,2} z^{m_{1,2}} (1-z)^{n_{1,2}} F_{1,2}(z)\,, \tag{3.20}$$

where $B_{1,2}$ are arbitrary constants $z = y^2$ and $F(z)$ are yet undetermined solutions. Substituting in the Dirac equations, separating the equations for ψ_1' and ψ_2' and demanding continuity of this solution with the solution obtained very close to the horizon, we finally obtain the following hypergeometric differential equations for $F_1(z)$ and $F_2(z)$:

$$\begin{aligned} & z(1-z)\frac{d^2F_i}{dz^2} + [(2m_i+1/2)-(2m_i+2n_i+3/2)z]\frac{dF_i}{dz} + \\ & -[m_i(m_i+1/2)+n_i(n_i+1/2)+2m_in_i - \frac{ilk_- - l^2k_-^2}{4}]F_i = 0\,. \end{aligned} \tag{3.21}$$

The constants m_i, n_i, the hypergeometric parameters $\alpha_i, \beta_i, \gamma_i$ and the integration constants B_i are tabulated below

$$\begin{aligned} m_1 &= \frac{1+ilk^+}{2} = m_2 + 1/2 \\ n_1 &= -\frac{1}{2}(L+1) = n_2 \\ \alpha_1 &= m_1 + n_1 + \frac{1}{2} + \frac{ilk_-}{2} = \alpha_2 + 1 \\ \beta_1 &= m_1 + n_1 - \frac{ilk^-}{2} = \beta_2 \\ \gamma_1 &= 2m_1 + \frac{1}{2} = \gamma_2 + 1 \\ B_2 &= -\left[\frac{\gamma-1}{\alpha-(2n+1)}\right] B_1\,. \end{aligned} \tag{3.22}$$

In our subsequent calculations, we shall normalise $B_1 = 1$. This solution is an exact solution for the BTZ space time and it approximates the fermionic wave function near the horizon.

The flux into the black hole can be calculated using the $\rho \to \rho_+, z \to 0$ limit of this solution. The flux of particles entering the horizon is

$$\mathcal{F}_0 = \sqrt{-g}J^\rho|_{\rho_+} = \sqrt{-g}\bar{\psi}e_1^\rho\gamma^1\psi\,. \tag{3.23}$$

Substituting the above solutions it is clear that ψ_2' dominates the flux, and the latter turns out to be

$$\mathcal{F}_0 = N \left| \frac{\gamma - 1}{\alpha - (2n+1)} \right|^2 . \tag{3.24}$$

Now, to find the incoming flux at infinity, we solve the radial Dirac equation in the 6-dimensional metric (2.1) away from the horizon, i.e. taking $r_n^2/r^2, r_0^2/r^2 \to 0$. Then the metric assumes the following form:

$$ds^2 = -\frac{1}{\sqrt{f_1 f_5}} dt^2 + \frac{1}{\sqrt{f_1 f_5}} dx_5^2 + \sqrt{f_1 f_5}\left(dr^2 + r^2 d\Omega^2\right). \tag{3.25}$$

The spin connections for this metric are:

$$w_t^{01} = -w_{x_5}^{51} = \frac{1}{4\sqrt{f_1 f_5}} \left[\frac{r_1^2}{r^3 f_1} + \frac{r_5^2}{r^3 f_5}\right], \qquad w_b^{i1} = \frac{1}{2} - \frac{r}{4}\left[\frac{r_1^2}{r^3 f_1} + \frac{r_5^2}{r^3 f_5}\right],$$

where b stands for the S^3 world indices and i the S^3 tangent space index. Now, in six dimensions the wavefunction is a four component chiral spinor. We start with the appropriate equation of motion in 6D as given in (2.8). Including all the terms, the equation of motion is:

$$\Big[(f_1 f_5)^{1/2}\,\Gamma^0 \partial_0 + \Gamma^1 \left(\partial_r + \frac{3}{2r} + \frac{1}{8} d_r(\ln(f_1 f_5))\right) + \\ + (f_1 f_5)^{1/2}\,\Gamma^5 \partial_{x_5} + \Gamma^b D_b\Big]\chi + g(r)\chi = 0 \tag{3.26}$$

Where $D_b = d_b + w_b$, where b denotes the S^3 directions, and $w_b^{ij}\sigma_{ij}$ is the spin connection with the i, j indices running over tangent space S^3 indices only. The function

$$g(r) = \frac{1}{12}\Gamma^{MNP} H_{MNP} = -\frac{1}{4}\left[d \ln(f_1 f_5)\right]\sqrt{\frac{f_1}{f_5}}.$$

Using the decomposition of Γ matrices into $\mathrm{SO}(2,1)$ and $\mathrm{SO}(3)$ parts we can separate out the equation of the components of the 6D chiral wavefunction into two sets of two component wave functions [25]. Again we expand in terms of the spherical harmonics on S^3 as: $\chi = \sum \chi'(x_\mu) Y$, where χ' are two component wave functions. Further, $\chi' = e^{i(\omega t - m\phi)}\,(f_1 f_5)^{-1/8}\, r^{-3/2}\psi''(r)$ is defined. Then from (3.26), we get:

$$\left[(f_1 f_5)^{1/2}\gamma^0 \partial_t + \gamma^1 \partial_r + (f_1 f_5)^{1/2}\gamma^2 \partial_{x_5}\right]\psi'' = \left[\frac{(L+3/2)}{r} - g(r)\right]\psi'' \tag{3.27}$$

Separating the components gives us the equations:

$$\left(d_r - (f_1 f_5)^{1/2}\imath\omega\right)\psi_1'' = \left(-\frac{L+3/2}{r} + g(r) - (f_1 f_5)^{1/2}\imath m\right)\psi_2'',$$

$$\left(d_r + (f_1 f_5)^{1/2}\imath\omega\right)\psi_2'' = \left(-\frac{L+3/2}{r} + g(r) + (f_1 f_5)^{1/2}\imath m\right)\psi_1''. \tag{3.28}$$

Defining, $\psi_1'' + \psi_2'' = (f_1 f_5)^{-1/4}\psi^+$ and $\psi_1'' - \psi_2'' = (f_1 f_5)^{1/4}\psi^-$ and with the additional approximation $g(r) = -1/4 d_r \ln(f_1 f_5)$, the equations reduce to:

$$\begin{aligned} \left(d_r + \frac{L+3/2}{r}\right)\psi^+ &= i(f_1 f_5)\omega\psi^- \\ \left(d_r - \frac{L+3/2}{r}\right)\psi^- &= i\omega\psi^+ \,, \end{aligned} \tag{3.29}$$

where we have put $m = 0$. The second order differential equation has the following form for ψ_-

$$\left[d_r^2 - \frac{(L+3/2)(L+1/2)}{r^2} + \omega^2(f_1 f_5)\right]\psi^- = 0\,. \tag{3.30}$$

We solve this equation in two regions: $r \sim l$ and $r \geq l$.

Intermediate region In the first region $r \sim l$, which we call the intermediate region, we take $\omega^2 f_1 f_5 \approx \omega^2(r_1^2 + r_5^2)/r^2 + \omega^2 l^4/r^4$ for low energy emissions. The differential equation in terms of $x = 1/r$ has the form:

$$\left[d_x^2 + \frac{2}{x}d_x - \frac{(L+3/2)(L+1/2) - (r_1^2 + r_5^2)\omega^2}{x^2} + \omega^2 l^4\right]\psi^- = 0\,. \tag{3.31}$$

The solution for the above differential equation is the Bessel function $x^{-1/2} Z_\nu(\omega l^2 x)$ where, $\nu = \sqrt{(L+1)^2 - (r_1^2 + r_5^2)\omega^2} \approx L+1$ for low energy emissions $\omega l \ll 1$. Hence explicitly the solutions are:

$$\psi^- = \sqrt{r}\left[a_1 J_{L+1}\left(\frac{\omega l^2}{r}\right) + a_2 N_{L+1}\left(\frac{\omega l^2}{r}\right)\right]. \tag{3.32}$$

And the coupled differential equation for ψ^+ yields:

$$\psi^+ = \frac{i l^2}{r^{3/2}}\left[a_1 J_L\left(\frac{\omega l^2}{r}\right) + a_2 N_L\left(\frac{\omega l^2}{r}\right)\right].$$

For $r < l$, the function $f \approx l^4/r^4$ and in the limit we are considering, i.e. $\omega l \ll 1, r \sim l$, we can do a small argument expansion of the bessel function. Hence

$$\begin{aligned} \psi_1'' + \psi_2'' &\approx \frac{il}{\sqrt{r}}\left[a_1 \frac{1}{L!}\left(\frac{\omega l^2}{2r}\right)^L + a_2 (L-1)!\left(\frac{2r}{\omega l^2}\right)^L\right], \\ \psi_1'' - \psi_2'' &\approx \frac{l}{\sqrt{r}}\left[a_1 \frac{1}{(L+1)}!\left(\frac{\omega l^2}{2r}\right)^{L+1} + a_2 L!\left(\frac{2r}{\omega l^2}\right)^{L+1}\right]. \end{aligned} \tag{3.33}$$

Which gives the leading order behavior of

$$\chi'_{1(2)} \sim a_2 L!\left(\frac{\omega l^2}{2}\right) r^{L-1/2}\,. \tag{3.34}$$

For $r > l$, $f_1 f_5 \approx 1$ and hence the above wavefunctions go to:

$$\psi_1'' + \psi_2'' \approx \frac{1}{r^{3/2}}\left[a_1 \frac{1}{L!}\left(\frac{\omega l^2}{2r}\right)^L + a_2 (L-1)!\left(\frac{2r}{\omega l^2}\right)^L\right],$$
$$\psi_1'' - \psi_2'' \approx \sqrt{r}\left[a_1 \frac{1}{(L+1)!}\left(\frac{\omega l^2}{2r}\right)^{L+1} + a_2 L!\left(\frac{2r}{\omega l^2}\right)^{L+1}\right]. \tag{3.35}$$

Which gives the wavefunction in the leading powers of r as:

$$\chi'_{1(2)} = a_2 L!\left(\frac{\omega l^2}{2}\right)^{L+1} r^L. \tag{3.36}$$

Far region. For $r > r_1, r_5$, we approximate $f_1 f_5 \approx 1$, and the second order differential equation for ψ^+ is:

$$d_r^2 \psi'^+ + \left[\omega^2 + \frac{(L+2)^2 - 1/4}{r^2}\right]\psi'^+ = 0. \tag{3.37}$$

This has the solution:

$$\psi'^+ = \sqrt{\omega r}\left(a_1' J_{L+2}(\omega r) + a_2' N_{L+2}(\omega r)\right). \tag{3.38}$$

Now, we can use this solution in the coupled equation (3.29) and get

$$\psi^- = \imath\sqrt{\omega r}\left[a_1' J_{L+1} + a_2' N_{L+1}\right].$$

We now see, how the wave functions behave and obtain matching conditions for their smooth joining. Using the expansion for Bessel functions we obtain the leading order behavior of the wavefunctions as: $r \sim l$:

$$\chi'_{1(2)} = a_1' \frac{\sqrt{\omega}}{(L+1)!}\left(\frac{\omega}{2}\right)^{L+1} r^L \tag{3.39}$$

and For $r \to \infty$ the asymptotic expansion of the bessel functions become important and the wavefunctions go as;

$$\chi'_{1(2)} = a_1' \frac{1}{\sqrt{2\pi r^3}} e^{-\imath\omega r}. \tag{3.40}$$

The flux at infinity entering the black hole spacetime is calculated from the asymptotic expansions of the Bessel functions, which is given by

$$\mathcal{F}_\infty = \frac{|a_1'|^2}{2\pi}. \tag{3.41}$$

Matching. To compare with the near horizon wave function solved in the x^+, r, x^- coordinates, we have to use the properties of the spinor under such transformations from t, r, ϕ coordinates. This gives a rotation on the two component wavefunction by a matrix :

$$\left[\cosh\left(\frac{\xi}{2}\right) + i\sinh\left(\frac{\xi}{2}\right)\sigma_2\right]\chi\,, \qquad \cosh\frac{\xi}{2} = \frac{\sqrt{\rho_+ + \rho_-}}{N^{1/4}} + \frac{\sqrt{\rho_+ - \rho_-}}{N^{1/4}}\,.$$

The near horizon solution, when extrapolated to $z \to 1$ (keeping the leading term in the expansion) is

$$\chi'_{1(2)} \to \sqrt{\rho_+ - \rho_-}L!\,2^{1/2}N^{-L/2}G\rho^{L-1/2}\,, \tag{3.42}$$

where

$$G = \frac{\Gamma(3/2 + i\omega/2\pi T_H)}{\Gamma([L+3]/2 + i\omega/4\pi T_+)\Gamma([L+2]/2 - i\omega/4\pi T_-)}\,,$$

Thus comparing with the intermediate solutions and then with the far solution using equations (3.34), (3.36) and (3.39) we get:

$$a'_1 = 2^{L+3/2}(L+1)L!^2\omega^{-L-3/2}N^{-(L/2)}G\,. \tag{3.43}$$

Substituting in F_∞ we finally get

$$\begin{aligned}\sigma_{\text{abs}} &= \frac{\pi(L+1)(L+2)}{\omega^3}\frac{F_0}{F_\infty} \\ &= \frac{\pi(L+2)N^{L+1}}{2(L+1)(L!)^4\,(\rho_+ - \rho_-)}\left(\frac{\omega}{2}\right)^{2L}\times \\ &\quad\times\cosh\left(\frac{\omega}{2T_H}\right)\left|\Gamma\left(\frac{L}{2}+\frac{1}{2}+\frac{i\omega}{4\pi T_+}\right)\Gamma\left(\frac{L}{2}+1+\frac{i\omega}{4\pi T_-}\right)\right|^2\,,\end{aligned} \tag{3.44}$$

where, we have used the fact that $|\Gamma(1/2+ix)|^2 = \pi/\cosh\pi x$ and we have multiplied by the appropriate plane wave normalisation [12]. The wavefunction corresponding to the S^3 spinor $Y^{p,-1/2}$ gives rise to a greybody factor with $T_+ \to T_-$ and vice-versa. Hence the total greybody factor is a sum of two terms, one due to each set of two component fermions.

3.3 Vector greybody factor

The vector equation of motion is given in (2.12). The higher partial wave in five dimensions gives a mass term for the gauge field in three dimensions. In addition, there is another set of equations as given in [27]:

$$\epsilon_\lambda{}^{\nu\rho}\partial_\nu A_\rho = -\frac{L}{l}A_\lambda\,. \tag{3.45}$$

This is derived from the representation theory of one forms on $\mathrm{SL}(2,\mathbb{R})$ manifolds. Since the BTZ space is locally anti-de Sitter, whose covering group is $\mathrm{SL}(2,\mathbb{R})\times$

$SL(2,\mathbb{R})$, the equation of motion gets supplemented by the above. On substituting the above equation in (2.12), the vector equation of motion reduces to (for convenience, we set the AdS radius $l = 1$ in the rest of this section):

$$\nabla^{\nu}\partial_{[\nu}A_{\lambda]} = L^2 A_\lambda \,. \tag{3.46}$$

It is to be observed that (3.46), can now be derived from (3.45) by operating with ∇ on both sides. There is also the consistency condition:

$$A^{\nu}_{;\nu} = 0\,. \tag{3.47}$$

We would like to solve the above equations of motion in the background of the BTZ black hole. In the coordinate system (μ, x^+, x^-) that we had adopted previously, the $+$ and $-$ components of (2.12) can be written as:

$$\partial^2 A_+ + (\tanh\mu - \coth\mu)\partial_\mu A_+ + 2\coth\mu\partial_+ A_\mu - 2\tanh\mu\partial_{[-}A_{\mu]} = L(L+2)A_+, \tag{3.48}$$

$$\partial^2 A_- - (\tanh\mu - \coth\mu)\partial_\mu A_- + 2\tanh\mu\partial_- A_\mu - 2\coth\mu\partial_{[+}A_{\mu]} = L(L+2)A_-, \tag{3.49}$$

where $\partial^2 \equiv g^{\alpha\beta}\partial_\alpha\partial_\beta = \partial^\mu\partial_\mu + \partial^+\partial_+ + \partial^-\partial_-$, we have taken $\epsilon^{+\mu-} = 1$ and have used the gauge condition (3.47). Defining

$$A_{1,2} = A_+ \pm A_- \tag{3.50}$$

it is clear that the equation for A_2 gets decoupled by adding (3.49) and (3.48). To decouple the equation for A_1, we use the equations (3.45) to substitute for the A_μ terms in (3.48) and (3.49). As a result, we get the following equations for the A_1 and A_2 (These set of equations can also be derived directly from (3.46):

$$\partial^2 A_i + (\tanh\mu + \coth\mu)\partial_\mu A_i = (L^2 - 2\epsilon_i L)A_1\,, \tag{3.51}$$

where $i = 1, 2$, $\epsilon_1 = -1$, $\epsilon_2 = 1$. Next, we substitute the solution

$$A_i = e^{ik_+x^+ + k_-x^-}A_i(\mu)\,,$$

which is consistent with the isometries of the metric. Substituting in (3.51), and defining $z \equiv \tanh^2\mu$ we get:

$$z(1-z)\frac{d^2A_i}{dz^2} + (1-z)\frac{dA_i}{dz} + \left[\frac{k_+^2}{4z} - \frac{k_-^2}{4} - \frac{L^2 - 2\epsilon_i L}{4(1-z)}\right]A_i = 0\,. \tag{3.52}$$

Next, we substitute the ansatz

$$A_i = e_i z^{m_i}(1-z)^{n_i}F_i(z)$$

in the above equation to obtain (e_i s are constants)

$$z(1-z)\frac{d^2F_i}{dz^2} + \Big[(1+2m) - z(1+2m+2n)\Big]\frac{dF_i}{dz} +$$

$$+\left[\frac{m^2 + k_+^2/4}{z} + \frac{n(n-1) - (L^2 - 2\epsilon_i L)/4}{1-z}\right]F - \left[(m+n)^2 + \frac{k_-^2}{4}\right]F = 0\,. \tag{3.53}$$

Continuity with the corresponding wave equations very close to the horizon ($z \to 0$) gives harmonic solutions in $\log z$ of the form $A_i = e_i^{in} e^{\imath k_1 logz} + e_i^{out} e^{-\imath k_2 \log z}$. To obtain a ingoing solution, we put $e_i^{out} = 0$. To ensure that (3.53) smoothly joins with this, we determine m and n and find that the coefficients of $1/z$ and $1/(1-z)$ terms vanish. The residual part of (3.53) is simply the hypergeometric differential equation. Thus the functions $F_i(z)$ are the hypergeometric functions $F[a_i, b_i; c_i; z]$ and the complete solution for the gauge potentials can be written as

$$A_i = e_i z^{m_i} (1-z)^{n_i} F[a_i, b_i; c_i; z] \,. \tag{3.54}$$

We can express the various parameters in terms of $k_\pm$ and L:

$$\begin{aligned}
m_i &= -i\frac{k_+}{2} \\
n_1 &= \frac{L}{2} + 1\,, & n_2 &= \frac{L}{2}\,, \\
a_1 &= -\frac{i}{2}(k_+ - k_-) + \frac{L}{2} + 1\,, & b_1 &= -\frac{i}{2}(k_+ + k_-) + \frac{L}{2} + 1\,, \\
a_2 &= a = -\frac{i}{2}(k_+ - k_-) + \frac{L}{2}\,, & b_2 &= b = -\frac{i}{2}(k_+ + k_-) + \frac{L}{2}\,, \\
c_i &= c = 1 + 2m_i\,.
\end{aligned} \tag{3.55}$$

$A_\pm$ can now be determined from the definitions (3.50) and the solution for A_μ can be constructed from the μ-component of (3.45):

$$A_\mu = \frac{1}{L\cosh\mu\sinh\mu}\partial_{[+}A_{-]}\,. \tag{3.56}$$

The important point to note is that the two components A_i satisfy equations which are scalar equations in the BTZ background. The spin dependence of the solutions is not obvious. The constants e_1 and e_2 are not independent by virtue of the auxiliary equations (3.45) and the consistency conditions. To determine the ratio, we use the equation with $\mu = +$ in (3.45).

$$-\tanh\mu\,(\partial_\mu A_- - \partial_- A_\mu) = LA_+\,. \tag{3.57}$$

On substituting A_μ from (3.45), and going the z coordinates, the equation reduces to in terms of A_1 and A_2 as,

$$\left[2zd_z - \frac{2k_-k_2}{L} + \frac{L}{1-z}\right] A_1 = \left[2zd_z - \frac{2k_-k_2}{L} - \frac{L}{1-z}\right] A_2\,. \tag{3.58}$$

On substituting the solutions for A_i, the above simplifies to:

$$\begin{aligned}
e_1 &\left[\frac{2abz}{c}F(a+1,b+1;c+1;z) + \left(a+b-\frac{2k_-k_2}{L}\right)F(a,b;c;z)\right] = \\
&= e_2(1-z)\left[\frac{2z(a+1)(b+1)}{c}F(a+2,b+2;c+1;z) + \right. \\
&\left. + \left(a+b+2-\frac{2k_-k_1}{L} - \frac{2(L+1)}{(1-z)}\right)F(a+1,b+1;c,z)\right].
\end{aligned} \tag{3.59}$$

Using a series of recursion relations, we get some simplified expressions [29]. The final expression is written below:

$$
\begin{aligned}
e_1 \left[2bF(a,b+1;c;z) + \left(a - b - \frac{2k_- k_2}{L}\right) F(a,b;c;z) \right] &= \\
= \frac{e_2}{a}[2(b-L)a + (a-b)L + 2k_- k_1] F(a,b+1;c;z) \\
+ \frac{e_2}{a}\left(a - b + 2k_- \frac{k_1}{L}\right)(a-L)F(a,b,c;z)\,. \qquad (3.60)
\end{aligned}
$$

From the above, the ratio of constants are now easily determined to be:

$$
\frac{e_2}{e_1} = -\frac{b^*}{a}\,, \tag{3.61}
$$

where $k_{1,2} \equiv [k_+ \pm k_-]/2$. Plugging in this ratio of constants into the solutions and using appropriate recursion relations, the wavefunctions can be written as

$$
\begin{aligned}
A_+ &= \frac{e_2}{2b^*}(1-z)^{L/2} z^{ilk^+/2} \left[-LF(a,b+1;c;z) + (L - ik^+)F(a,b;c;z) \right] \\
A_- &= -\frac{e_2}{2b^*}(1-z)^{L/2} z^{ilk^+/2} \left[LF(a,b+1;c;z) + ik^- F(a,b;c;z) \right] \qquad (3.62)
\end{aligned}
$$

In the above, the solution is actually the real part of the wave function determined above. The flux of the vector field at the horizon of the black hole is calculated using the energy momentum tensor for the massive vector field. Since our wave function is Re A_i, the energy momentum tensor which involves products of the fields will have the square terms proportional to $e^{2i\omega t}$ and $e^{-2i\omega t}$. Under time averaging, these terms go to zero, and hence the steady rate of particle influx is given by cross terms :

$$
T_{\nu\lambda} = -\frac{1}{4}(|F|^2 + 2m^2|A|^2)g_{\nu\lambda} + F_{\nu\sigma}F^{*\ \sigma}_{\lambda} + m^2 A_\nu A^*_\lambda\,, \tag{3.63}
$$

where m stands for the mass. For our purposes $m^2 = L(L+2)$. To determine the flux, we incorporate the red-shift factor and integrate over the horizon area to get:

$$
\mathcal{F}_0 = \frac{l^2 N^2 L}{2\rho_+} k_+^2 \left|\frac{e_2}{b}\right|^2 \Omega\,, \tag{3.64}
$$

where $N \equiv \rho_+^2 - \rho_-^2$, $k^+ = \omega/(2\pi l T_H)$, and we have restored the radius of anti-desitter space. Also $\Omega = 8\pi^2$ denotes the factors which come from the angular integrals. Note that the flux vanishes for $L = 0$, since the latter is a not a dynamical mode [27].

Before determining the waveform at infinity, we solve (3.46) in the asymptotic AdS_3 metric in the coordinates (t, ρ, ϕ)as an interesting exercise, as it sheds light on the boundary behavior of the wavefunction in the BTZ geometry. The wavefunctions, $A_i = e^{i\omega t} B_i$ are solved, with the help of (3.45) as:

$$
B_i(x) = \sqrt{\rho}\left[c_i J_{\nu_i}\left(\frac{\omega l^2}{\rho}\right) + d_i N_{\nu_i}\left(\frac{\omega l^2}{\rho}\right)\right], \tag{3.65}
$$

where J_ν and N_ν are Bessel functions of the first and second kind respectively, $\nu_1 = L-1$, $\nu_2 = L+1$ and c_i, d_i are arbitrary constants. Further, consistency with the equations (3.45) requires that $c_1 = -c_2 \equiv c$ and $d_1 = -d_2 \equiv d$.

To determine the wavefunction at asymptotic infinity which joins with the BTZ wavefunction, we need to look at the vector equation of motion in six dimensions. As given in eq. (2.11), the vector equation of motion involves all the other A_a components which are scalar in the t, r, ϕ plane as well as the H_{MNP} three form field strength in six dimensions. Since we are interested in that part of GBF which is due to the three dimensional vectors, we take $N = \mu$ in eqn 2.11 and take the limit $r \to \infty$. The equation of motion reduces to:

$$\nabla^\nu F_{\mu\nu} + \nabla^a F_{a\mu} = 0\,, \tag{3.66}$$

where we have kept terms of $\mathrm{O}(1/r^2)$. In six dimensions $H_{MNP} = \epsilon_{ijkl} d_l f_5$ where ϵ_{ijkl} is the flat space epsilon tensor along the four non-compact directions x_i, which gives the second term in equation (2.11) to be order $(1/r^3)$ form (2.2) and hence can be ignored. In the gauge $\nabla^M A_M = 0$, we assume that $\nabla^\nu A_\nu = \nabla^a A_a = 0$. The main observation is that the A_a's decouple in this gauge. For the wavefunctions $A_\mu = e^{i\omega t} e^{im\phi} A'_\mu(r)/r^{3/2}$, the equation of motion for the $m = 0$ case is of the form:

$$\partial_r^2 A'_{t,\phi} + \left[\omega^2 - \frac{(L+1)^2 - 1/4}{r^2}\right] A'_{t,\phi} = 0\,. \tag{3.67}$$

The solutions are:

$$\begin{aligned}
A'_t &= \frac{1}{r}\left[a_1 J_{L+1}(\omega r) + a_2 N_{L+1}(\omega r)\right] \\
A'_\phi &= \frac{1}{r}\left[a'_1 J_{L+1}(\omega r) + a'_2 N_{L+1}(\omega r)\right] \\
A'_r &= \frac{1}{r^3}(-\imath\omega) \int_r^\infty r'^3 A_t(r') dr'\,.
\end{aligned} \tag{3.68}$$

It is interesting to note that the wavefunctions determined here do not share the exact polynomial nature of the wavefunction obtained in (3.65) at $r = \rho = l$, as in the case of scalars. The reason behind this is that due to the loss of $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ symmetry, the equations (3.45) are no longer valid for the asymptotic metric. Thus the wavefunctions match with each other only in leading order in ωr. Let us find the relation between the coefficients of the solutions (3.54) and (3.68) for which, we compare the two solutions in the region $z \to 1$, and $r\omega \ll 1$. Using standard results for the behaviour of hypergeometric functions as $z \to 1$, we find the leading behaviour of the wave functions as [30]:

$$A_+, A_- \to \frac{e_2}{2b^*} \frac{(N)^{-L/2} L \Gamma(L) \Gamma(c)}{\Gamma(a)\Gamma(b+1)} \rho^L\,. \tag{3.69}$$

We match the solutions with the far region wavefunctions using the relation: $A_t l = \rho_+ A_+ - \rho_- A_-$ which gives:

$$a_1 l = a_1' = \frac{e_2}{2b*} (\rho_+ - \rho_-) N^{-L/2} \left(\frac{\omega}{2}\right)^{-(L+1)} \Gamma(L+2)\Gamma(L+1)E_1 \tag{3.70}$$

Where $E_1 = \Gamma(c)/(\Gamma(a)\Gamma(b+1))$. The other constants are negligible and hence ignored. The solutions go as $A_i' \sim \sqrt{1/2\pi\omega} e^{-i\omega r}$ at large distances. The flux, determined from equation (3.63) is:

$$\mathcal{F}_\infty = -\frac{\omega}{2\pi} l^2 |a_1|^2 \Omega \, . \tag{3.71}$$

Taking the ratio of the near horizon and asymptotic fluxes (3.64) and (3.71) and using the above relations for the ratio of the constants, we finally get the probability of absorption of the L^{th} partial wave as

$$\mathcal{P}_L = \frac{\mathcal{F}_0}{\mathcal{F}_\infty} = \frac{\pi L k^{+2} N^L \omega^{2L+1}}{l^2 \rho_+ 2^{2L} (\rho_+ - \rho_-)^2 (\Gamma(L+2)\Gamma(L+1))^2 |E_1|^2} \, . \tag{3.72}$$

This is the general result for the partial wave L. It is clear that the evaluation of the gamma-functions will give rise to the familiar form of the greybody factor with thermal distribution functions corresponding to two incoming particles and one outgoing particle. The latter always is always associated with a Bose distribution function, as can be seen from the relation $|\Gamma(c_1)|^2 = |\Gamma(1 + \omega/2\pi T_H)|^2 = (\omega/2T_H)/\sinh(\omega/2T_H)$. However, the nature of the 'ingoing' distribution functions depend on the value of L that one considers. In particular, on substituting the values of a and b from (3.55) in $\mathcal{P}$, we find that the the gamma-functions in the numerator correspond to Fermi distributions for odd-L and bose distributions for even-L. Thus, depending on the partial wave, the vector particle can be thought of arising out of the interactions of two bosons or two fermions.

The greybody factor or the absorption coefficient of the black hole is determined by multiplying by the plane wave factor as:

$$\begin{aligned} \sigma_{\text{abs}} = {} & \frac{2 L N^{L+2}}{(L!)^4 2^{2L}} \frac{\omega^{2L}}{l^2 \rho_+ T_H (\rho_+ - \rho_-)^2} \times \\ & \times \frac{\sinh \omega/T_H}{\omega} \left| \Gamma\left(\frac{L}{2} + \frac{i\omega}{4\pi T_+}\right) \Gamma\left(\frac{L}{2} + 1 + \frac{i\omega}{4\pi T_-}\right) \right|^2 . \end{aligned} \tag{3.73}$$

If we include rest of the components of the six dimensional vector, i.e. A_a, then the total GBF will involve a sum of the individual greybody factors. The greybody factors due to A_a are same as that of the scalars. Since those terms do not contain the spin dependence, we ignore them.

4. CFT description

The decay rates are obtained from the above greybody factors by multplying with the appropriate Planck or Fermi-Dirac distributions. It has been known for long that the these decay rates can be reproduced form a CFT calculation using appropriate conformal operators. Earlier, the dimensions of the CFT operators were guessed from the structure of the decay rates [32, 33]. However, using the AdS/CFT correspondence, the dimension as well as the exact correlators with correct normalisations can be determined using prescriptions given in [13, 14, 15, 16, 20]. Here we rely on the correspondence to determine the correlators. We note that the near horizon approximation of the black holes can be used at most till $r \sim l$. Though the near horizon metric will receive corrections as r approaches l, we ignore them in this region. The correlators are determined in Poincare coordinates for convenience.

The Poincare coordinates are related to the BTZ coordinates by the following relations:

$$w^{\pm} = \left(\frac{\rho^2 - \rho_+^2}{\rho^2 - \rho_-^2}\right)^{1/2} e^{2\pi T_{\pm}(t \pm \phi l)}\,, \qquad x_0 = \left(\frac{N}{\rho^2 - \rho_-^2}\right)^{1/2} e^{\pi T_+(t+\phi l) + \pi T_-(t - \phi l)}\,.$$

The metric in Poincare coordinates is:

$$ds^2 = \frac{l^2}{x_0^2}\left(dx_0^2 + dw^+ dw-\right) \tag{4.1}$$

The Klein-Gordon equation on this background can be written in the following form:

$$\left[\partial_{x_0}^2 - \frac{1}{x_0}\partial_{x_0} + 4\partial_+\partial_- - \frac{L(L+2)}{x_0^2}\right]\phi = 0 \tag{4.2}$$

Substituting:

$$\phi = \int d^2 w \phi_k(x_0) e^{i\vec{k}.\vec{w}}\,. \tag{4.3}$$

The solutions which are ingoing or regular at the black hole horizon are:

$$\phi_k(x_0) = a x_0 K_{L+1}(k x_0)\,,$$

where, $k = 4k_+k_-$ and a is an arbitrary constant of integration. To determine the correlator corresponding to the above scalar field and look at the bahavior of the wavefunction at $r = l$, which implies $x_0 \sim r_0/l \approx 0$ in the dilute gas approximation. The boundary of the AdS field is taken at $x_0 = \epsilon$ where ϵ is infinitesimally small and set set $\phi_k(\epsilon) = 1$. The action is:

$$I = \frac{1}{2}\int d^2 w dx_0 \frac{1}{2x_0^3}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right]. \tag{4.4}$$

On partially integrating, the boundary term from this action at $x_0 = \epsilon$ is:

$$I_B = \int d^2w \frac{1}{2\epsilon} \lim_{x_0\to\epsilon} \phi d_{x_0}\phi \tag{4.5}$$

On using (4.3) in the above, and using the solutions for $\phi_k(x_0)$, the action (Fourier component) consists only of the boundary term at $x_0 = \epsilon$. The Fourier component thus is:

$$\lim_{x_0\to\epsilon} \epsilon^{-1}\delta(k+k')\frac{K_{L+1}(kx_0)}{K_{L+1}(k\epsilon)} d_{x_0} \frac{x_0 K_{L+1}(kx_0)}{\epsilon K_{L+1}(k\epsilon)}\,,$$

Using the expansion for

$$\begin{aligned} K_n(kx_0) &= \frac{1}{2}\sum_{k=0}^{k=\infty}(-1)^k\frac{(n-k-1)!}{k!}\left(\frac{z}{2}\right)^{2k-n} + \\ &+ (-1)^{n+1}\sum_{k=0}^{k=\infty}\frac{1}{k!(n+k)!}\left(\frac{z}{2}\right)^{n+2k}\left[\ln\frac{z}{2} - \frac{1}{2}\Psi(k+1) - \frac{1}{2}\Psi(k)\right], \end{aligned} \tag{4.6}$$

the expression reduces to:

$$\frac{2(L+1)}{(L+1)!\,L!}\left(\frac{k}{2}\right)^{2L+2}\epsilon^{2L+1}\ln\frac{k\epsilon}{2}$$

The leading non-analytic term has a $\ln(k\epsilon)$ dependence. We keep the coefficient of the term with ϵ dependence as $\epsilon^{2L+1}\ln\epsilon$ and Fourier transform to position space to get the correlator

$$G^s(w,w') = 2(L+1)^2\frac{1}{|\vec{w}-\vec{w'}|^{2L+2}} \tag{4.7}$$

For the fermionic correlators, we do a calculation similar to that done in [16, 18]. The boundary is taken at ϵ. The action is taken as:

$$I = \int d^2w dx_0 \frac{1}{2x_0^3}\bar{\psi}(\nabla\!\!\!/ + L + 1/2)\psi + C\int d^2w\bar{\psi}\psi\,, \tag{4.8}$$

where C is a constant, which gets fixed when we try to obtain exact matching. The solution of the equation of motion in the representation of γ matrices where γ_0 is diagonal, the two components of ψ are:

$$\psi_1 = \int d^2w e^{i\vec{k}.\vec{w}} a_1 K_L(kx_0)\,, \qquad \psi_2 = \int d^2w e^{i\vec{k}.\vec{w}}\frac{i\gamma.\vec{k}}{k}K_{L+1}(kx_0)\,. \tag{4.9}$$

On specifying one of the components at $x_0 = \epsilon$, the other component also gets related to it. Using the above solutions, and substituting in the boundary term of the action, one Fourier component is read as:

$$\lim_{x_0\to\epsilon}\delta(k+k')\frac{\vec{k}\cdot\gamma_{\alpha\beta}}{k}\frac{K_L(kx_0)}{K_{L+1}(k\epsilon)}$$

Taking the expansion for K_n as given in equation (4.6), and keeping the coefficient of the $\epsilon^{2L+1}\ln\epsilon$ term, the greens function in the Fourier transformed space is read off as:

$$G^f(w,w')_{\alpha\beta} = (L+1)\frac{(\vec{w}-\vec{w'})\cdot\gamma_{\alpha\beta}}{|\vec{w}-\vec{w'}|^{2(L+2)}} \tag{4.10}$$

Where $\alpha\beta$ stand for spinor indices. The correlator has also been determined in [18].

To find out the correlators for the CFT operators corresponding to the vectors, it is useful to employ the methods of [16, 19]. We solve the vector equations in AdS_3 space, in Poincare coordinates. The equation of motion for $A_0 = \int d^2we^{i\vec{k}.\vec{w}}A_0$ and $A_i = \int d^2we^{i\vec{k}.\vec{w}}A_i$ have the forms:

$$\begin{aligned} d^2_{x_0}A_0 - \frac{1}{x_0}d_{x_0}A_0 - \left(k^2 + \frac{L^2-1}{x_0^2}\right)A_0 &= 0\,, \\ d^2_{x_0}A_\pm + \frac{1}{x_0}d_{x_0}A_\pm - \left(k^2 + \frac{L^2}{x_0^2}\right) &= \frac{2}{x_0}ik_\pm A_0\,. \end{aligned} \tag{4.11}$$

The equation for A_0 is easily solved as $A_0 = a_0x_0Z_L(kx_0)$, where $k^2 = 4k_+k_-$. For $k^2 > 0$, $Z_m(kx_0) = K_m(kx_0)$ (modified Bessel function of second kind) and for $k^2 < 0$, this is $Z_m(kx_0) = J_m(kx_0)$. However, since we confine ourselves to Euclidean metric, we choose the former solution. The other two components are easily separated using equation (3.45). The solutions for the two components are:

$$A_\pm = a_\pm x^0 K_{L\pm1}(kx_0)\,. \tag{4.12}$$

The use of (3.45) also leads to a relation between the constants $a_i's$, which implies, that only one can be fixed independently by boundary condition. The other arbitrary constants are related to it, and hence are determined. Thus only one component of A_i can be fixed at the boundary, and hence the classical source is actually chiral. The ratio of constants are as follows:

$$a_\pm = a_0\frac{k_\pm}{k}\,. \tag{4.13}$$

This obviously implies that $a_+/a_- = k_+/k_-$. Also, the function A_- falls slower than A_+, and hence we specify A_- at the boundary. The other components then get related to it. The expression for the action is:

$$I = \int d^2wdx_0\frac{1}{2x_0^3}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}L(L+2)A_\mu A^\mu\right]. \tag{4.14}$$

The boundary term which comes due to one partial integration is:

$$I_B = \int d^2w\frac{x_0}{2}\left[A_+F_{0-} + A_-F_{0+}\right]. \tag{4.15}$$

Using the solutions obtained above, one Fourier component of the action evaluated at a distance ϵ is:

$$I = \delta(k+k')\epsilon \frac{K_{L+1}(k\epsilon)}{K_{L+1}(k\epsilon)} \frac{k_- K_{L-1}(k\epsilon)}{k_+ K_{L+1}(k\epsilon)} \,. \tag{4.16}$$

On using the expansion of (4.6) we find:

$$\begin{aligned} \frac{K_{L-1}}{K_{L+1}} &= \frac{(L-2)!\left(\frac{k\epsilon}{2}\right)^{1-L} + \cdots + \frac{(-1)^L}{(L-1)!}\left(\frac{k\epsilon}{2}\right)^{L-1}\ln(\frac{k\epsilon}{2})}{L!\left(\frac{k\epsilon}{2}\right)^{-1-L} + \cdots + \frac{(-1)^{L+2}}{(L+1)!}\left(\frac{k\epsilon}{2}\right)^{L+1}\ln(\frac{k\epsilon}{2})} \\ &= \frac{(-1)^L \ln(k\epsilon/2)}{L!}\left[\frac{1}{(L-1)!}\left(\frac{k\epsilon}{2}\right)^{2L} - \frac{(L-2)!}{L!\,(L+1)!}\left(\frac{k\epsilon}{2}\right)^{2L+2}\right] . \end{aligned} \tag{4.17}$$

Thus retaining only the leading power of ϵ in the above and using that in (4.16), we get:

$$I = \epsilon^{1+2L} \ln\epsilon \frac{k_-^2 k^{2L-2}}{2^{2L-2} L!\,(L-1)!}\delta(k+k') \,. \tag{4.18}$$

The Fourier transform of this yields in Poincare coordinates, the correlation function as:

$$\langle O^v O'^v \rangle = 2(L+1)\frac{(w^+ - w'^+)^2}{|\vec{w'} - \vec{w}|^{2(L+1)+2}} \,. \tag{4.19}$$

We now calculate the emission rates from the conformal field theory theory. The quantum mechanical calculation involves modelling the entire black hole spacetime by a CFT at the boundary of the near horizon geometry $r \sim l$. This is in accordance with the AdS/CFT correspondence, as the information about the near horizon $BTZ \times S^3$ is supposed to be encoded in the boundary of the BTZ space. A plane wave is taken to be incident on the black hole, which couples with the operators of the CFT in the region $r \sim l$. The emission rate due to this excitation is calculated using the results for ordinary stimulated emission. The incident wave is regarded as classical, while the CFT operators are treated as quantum. The plane wave has to be expanded in spherical waves, to get out the partial wave components. The plane wave is expanded in terms of spherical functions as:

$$e^{ikx} = \sum_{L\geq 0} \sqrt{2\pi^2}(L+1)\frac{e^{-\imath\omega r}}{(r\omega)^{3/2}} e^{\imath\psi} Z_{l,0}(cos\theta) \,. \tag{4.20}$$

The spherical wave, near $r = l$ goes as $r^n \phi_0(\phi, t)$ (where n is an integer depending on L). It couples to the CFT operator as $\int d^2x \phi_0 \mathcal{O}$.

To determine the dimension of ϕ_0 under conformal transformations, we look at the behaviour of the wave function at $r \sim l$. The part of the wave function which goes as r^n comes from contribution of $J_\nu(\omega r)$ which is analytic in the region we are considering. For the scalars, the wavefunction near $r \sim l$ goes as $r^L \phi_0$. As on the boundary, the theory is invariant under conformal transformations, ϕ_0 should have a definite behavior under transformations which can be scalings like $ds^2 = f^2(r)(ds'^2)$. Here the coordinates scale like $f(r)$ and the wavefunction scales like $\phi = f^L r^L \phi'_0$. Since ϕ is a scalar, ϕ_0 has to scale as f^{-L}. Which gives it a dimension of $-L$. Accordingly, the coupling $\int d^2x\phi_0\mathcal{O}$ implies the dimension $\Delta_S = L+2$ for the operator O. This is consistent with the correlator determined earlier.

For the fermions, the two components of the wave function do not fall off in an identical manner, and the eigenstates of γ^1 (which is the chirality matrix for two dimensions), $\chi_1+\chi_2 = \chi_+$ falls slower than $\chi_1-\chi_2 = \chi_-$ as given in equation (3.34). So for our purposes, we take $\psi_+ \approx 0$. For

$$\chi_- = \left(\frac{1}{r}\right)^{1/2-L} \psi_0(t,\phi)$$

and we get the fall off power of χ_- as $\lambda = L-1/2$. Since the fermion is a scalar under transformations $r' = f(r)r$, ψ_0 has the dimension of $L-1/2$ under this transformation, which is like a conformal transformation in the boundary metric. Hence by conformal invariance of the term $\int d^2x\phi_0\mathcal{O}$, the dimension of O is $\Delta_F = 2+\lambda = L+3/2$. The operator ψ_0 is a spin 1/2 object under the group SO(2,2), and hence, O is also spin-1/2, but of opposite chirality. Hence, the left and right conformal weights are determined as $h_- + h_+ = L+3/2$ and $h_- - h_+ = 1/2$, thus

$$h_- = \frac{L}{2}+1, h_+ = \frac{L}{2}+\frac{1}{2}\,.$$

This is the same as that appears from the correlator calculation given above.

As for the vector field, it is immediately observed, that the two separable components at the boundary, $A_{1,2} = A_t \pm A_\phi/l$, correspond to left moving and right moving sources in the boundary. The fall off in powers of r is different for the two components and the case we are considering, and as seen from earlier section, A_1 falls slower than A_2, and hence $A_2 \approx 0$. The fall off in A_1 is as follows:

$$A_1 = r^L A_0(t,\phi)\,.$$

Since under the transformation $r \to f(r)r$, A_1 transforms as a covariant vector, the dimension of A_0, $\lambda = L-1$. Thus A_0 is a source for the CFT operator O_v with weight $\Delta_v = 2+L-1 = L+1$. The left and right weights can now be determined as

$$h_- = \frac{L}{2}+1\,, \qquad h_+ = \frac{L}{2}\,.$$

All the weights determined above are same as those predicted using group theoretic methods in [27].

We are now ready to compute the emission rate due to the plane wave-CFT coupling $\int \phi_0 O$. If due to this interaction term, the state in the CFT undergoes a transition, $|i\rangle \to |f\rangle$, then, the transition probability for this process is:

$$w_{fi} = |\phi_0|^2 |\langle f|\, O\,|i\rangle|^2 \delta(\epsilon_f - \epsilon_i - \omega)\,,$$

where ϵ_f, ϵ_i are the energies at of the initial and final states of the CFT. The above can be written as an integral over the two coordinates of the boundary, and in case the final state is not a unique state, we sum over the final states which gives:

$$T = \sum_f \int d^2x e^{i\omega t} \left\langle i|e^{i\epsilon_i t}O^\dagger e^{-i\epsilon_f t}|f\right\rangle \langle f|O|i\rangle \, |\phi_0|^2 \,.$$

If the initial state is the Poincare vacuum, then the transition probability is:

$$T = \int d^2x e^{i\omega t} \left\langle 0|O^\dagger(t)O(0)|0\right\rangle |\phi_0|^2 \,. \tag{4.21}$$

Essentially we need $G(w,w')|\phi_0|^2$ to complete the calculations. For ϕ_0 we use the form of the plane wave solutions at $r \sim l$ as determined in section 3. However, as these have been determined in BTZ coordinates, we use the conformal dimension of these when we use Poincare coordinates. In effect,

$$\phi_0^P = (2\pi T_+ w^+)^{h_+-1}(2\pi T_- w^-)^{h_--1}(Nl^2)^{h_++h_-}\phi_0^{BTZ} \,.$$

An additional power of $(Nl^2)^{h_++h_-} = (4\pi^2 T_- T_+ l^4)^{h_++h_-}$ enters, since in BTZ coordinates, we assume that the wave function scales like $r^{h_++h_-}$ at the boundary and in the Poincare coordinates $x_0^{-(h_++h_-)}$. ($x_0 = \sqrt{N}/r$ at the boundary, and we use l to make the scalings in both the coordinates dimensionless) Using the fact that $t = (1/4\pi T_+)\ln w^+ + (1/4\pi T_-)\ln w^-$, we get the integral in the transmission coefficient to be:

$$I = \int dw^+ dw^- \frac{(w^+)^{i\omega/4\pi T_+ + h_+ -1}(w^-)^{i\omega/4\pi T_- + h_- -1}}{(w^+-1)^{2h_+}(w^--1)^{2h_-}} \,,$$

where we take the initial

$$w'^{\pm} = e^{2\pi T_\pm (t\pm\phi)}$$

at the origin of the BTZ coordinates. The range of $w^\pm$ is from 0 to ∞. Changing from $w^\pm \to -w^\pm$, and using $B(x,y) = \int_0^\infty dt\ t^{x-1}/(1+t)^{x+y}$, the integral can be done:

$$I = \frac{1}{\Gamma(2h_+)\Gamma(2h_-)} e^{-\omega/2T_H} \left|\Gamma\left(h_+ + \frac{i\omega}{4\pi T_+}\right)\right|^2 \left|\Gamma\left(h_- + \frac{i\omega}{4\pi T_-}\right)\right|^2 \,.$$

The emission rate is evaluated as:

$$(2\pi l^2 T_+)^{2h_+-1}(2\pi l^2 T_-)^{2h_--1}|\phi_0|^2 C I \,,$$

where C is a normalisation constant, which includes the plane wave normalisation. Plugging in the correct normalisation for each of the correlators, and using the appropriate ϕ_0, the emission rates are exactly same as the semiclassical calculations. For the scalars, $\phi_0 = 1/(L+1)!\,(\omega/2)^{L+1}$. Using this, as well as $h_- = h_+ = L/2+1$, the relation that $4l^4\pi^2 T_+ T_- = N$,and multiplying by the appropriate factor to get the plane wave normalisation, we get the emission rate as:

$$\Gamma^S_{\rm cft} = 2\pi N \frac{(N\omega^2)^L}{2^{2L}(L!)^4} \frac{\exp(-\omega/2T_H)}{\omega} \left| \Gamma\left(\frac{L}{2}+1+\frac{i\omega}{4\pi T_+}\right) \Gamma\left(\frac{L}{2}+1+\frac{i\omega}{4\pi T_-}\right)\right|^2 . \tag{4.22}$$

A comparison with equation (3.14), shows that the semiclassical calculation has been reproduced exactly. There is an alternative derivation for the s-wave emission in [34].

For the fermions, the wave is chosen to be of a given chirality, and hence in the expression for the emission rate, $\psi_0 = \omega^{L+3/2} 2^{-(L+1)}/(L+1)!$ with $h_- = L/2+1, h_+ = L/2+1/2$, the emission rate is determined as:

$$\begin{aligned}\Gamma^F_{\rm cft} = & \frac{\pi(L+1)^2(L+2)(2\pi l^2 T_-)^{L+1}(2\pi l^2 T_+)^L}{4(L+1)!^2\Gamma(L+1)\Gamma(L+2)} \left(\frac{\omega}{2}\right)^{2L} \times \\ & \times e^{-\omega/2T_H} \left| \Gamma\left(\frac{L}{2}+1+\frac{i\omega}{4\pi T_-}\right)\left(\frac{L}{2}+\frac{1}{2}+\frac{i\omega}{4\pi T_+}\right)\right|^2 . \end{aligned} \tag{4.23}$$

Using the expressions for the temperatures, it can be seen that the above expression exactly matches that obtained in equation (3.44) after multiplying by the Fermi-Dirac distribution, $1/(\exp(\omega/T_H)+1)$. The special case of s-waves for $T_- \gg T_+$ was obtained in [35]. The GBF for the other set of two component wave functions in six dimension can be obtained by the same procedure above, but now with h_+ and h_- interchanged.

For the vector coupling, we retain the component of the wave function which falls slower as a function of r at $r \sim l$. This couples to the operators on the boundary. Hence for the vector $\phi_0 = A_t + A_\phi = 1/(L+1)!\,(\omega/2)^{L+1}$. This along with $h_- = L/2+1, h_+ = L/2$, yields the emission rate as:

$$\begin{aligned}\Gamma^V_{\rm cft} = & 2\pi \left(\frac{\omega}{2}\right)^{2L} \frac{(L+1)^3}{\Gamma^2(L+2)} \frac{(2\pi l^2 T_+)^{L+1}(2\pi l^2 T_-)^{L-1}}{\Gamma(L)\Gamma(L+2)} \times \\ & \times \frac{e^{-\omega/T_H}}{\omega} \left| \Gamma\left(\frac{L}{2}+\frac{i\omega}{4\pi T_+}\right) \Gamma\left(\frac{L}{2}+1+\frac{i\omega}{4\pi T_-}\right)\right|^2 \\ = & 2\pi \left(\frac{N\omega^2}{4}\right)^L \frac{NL}{(L!)^4(\rho_+-\rho_-)^2} \times \\ & \times \frac{e^{-\omega/2T_H}}{\omega} \left| \Gamma\left(\frac{L}{2}+\frac{i\omega}{4\pi T_+}\right) \Gamma\left(\frac{L}{2}+1+\frac{i\omega}{4\pi T_-}\right)\right|^2 . \end{aligned} \tag{4.24}$$

This is same as eqn (3.73) multiplied by the Planck distribution with temperature T_H. Thus we see for each of the cases stated above the matching is exactly obtained. It is interesting to note how the various factors conspire among themselves to yield this exact matching, using the AdS/CFT correspondence.

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5. Discussions

In this paper, we have studied the emission rate for particles for arbitrary partial waves by probing the near horizon geometry of a 5-dimensional near extremal black hole. We determined the greybody factors of scalars, spinors and vector particles by solving their respective equation of motion in the BTZ back ground and matching them with wavefunctions obtained at large distances from the black hole. For fermions, the matching was non-trivial, and we solved the equation of motion in an intermediate region; $r \sim l$. The answers obtained for the scalars and spinors reproduced the results obtained previously for the five dimensional black hole. Our calculation for non-minimally coupled vector particles is the first calculation for emission rates for the given configuration.

Next, we used the conformal field theory at the boundary to obtain the quantum mechanical spontaneous emission rates. This is in the spirit of the AdS/CFT correspondence, in which all information regarding the bulk degrees of freedom are entirely encoded in the degrees of freedom at the boundary. Indeed, we used the various 2-point functions which have been calculated from the AdS/CFT correspondence to find the decay rates, and the latter perfectly matches with the semi classical Hawking radiation rates, *for all partial waves*. The asymptotic plane waves that excite the CFT near $r \sim l$ carry non-trivial kinematical information and influence the spontaneous emission rate. Thus our calculation shows how the AdS/CFT correspondence can be successfully used to predict the emission rates from black holes. The exact matching suggests that the thermodynamical properties of these black holes are 'holographically' encoded in the boundary CFT.

It is to be noted that the CFT is at a finite distance from the horizon, and the role of the horizon degrees of freedom are not very clear, unlike the CFT determined in [36]. Also the thermodynamics of non-extremal black holes like the Schwarzschild black hole remains unaddressed, as the near horizon $BTZ \times S^3$ geometry emerges only for near extremal black holes.

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